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A Hail Size Forecasting Technique

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ABSTRACT

A technique for forecasting the size of hailstones accompanying thunderstorms is presented. Hailstone size is related to its terminal velocity which in turn is related to the updraft velocity of a thunderstorm as derived from parcel buoyancy. This updraft velocity is approximated from positive area measurements on a thermodynamic diagram. The technique is tested on proximity soundings taken near the site and prior to known hail occurrences.

INTRODUCTION

THE best method currently available for forecasting hailstone size is that developed by Fawbush and Miller [1]. Their method relates hailstone size found at the Earth's surface to a measure of the energy available in the form of hydrostatic instability of the airmass within which the hail-producing thunderstorms occur. The use of their method involves analysis of soundings or predicted soundings near an area of expected thunderstorm activity. This analysis, carried out on thermo-dynamic diagrams, yields positive areas which are proportional to the energy available from hydrostatic instability. Although based on the above reasoning the development of their method was largely empirical. Some of the dimensioning parameters may, at times, be artificial, or may be more related to the probability of thunderstorm occurrence than to hailstone size. Nevertheless, results of the Fawbush and Miller method were encouraging, so encouraging that a similar method has been developed, but more along a physical basis. Such a method should be more amenable to diagnostic analysis for weak points, and would offer promise for further development. The object of this study was to develop such a technique.

A theory relating hailstone size to updraft velocity evolved early in our science. The observed concentric, spherical, laminar structure of the ice comprising the hailstone led first to the theory of formation during repeated traverses of the freezing level by a hailstone "bobbing" in a strong updraft [2].

The alternative theory presented by Schumann [3], Gaviola and Fuertes [4], and others, which proposes growth during true fall must, nevertheless, require updrafts to retard the hailstone in the zone of formation in order to permit growth to the sizes observed.

It is assumed here that the velocity required to retard the hailstone in its fall might not be very different from that required to suspend it. The technique described here is based upon the following premises:

(1) that the updraft velocity prevailing in the zone of hail formation is the velocity required to

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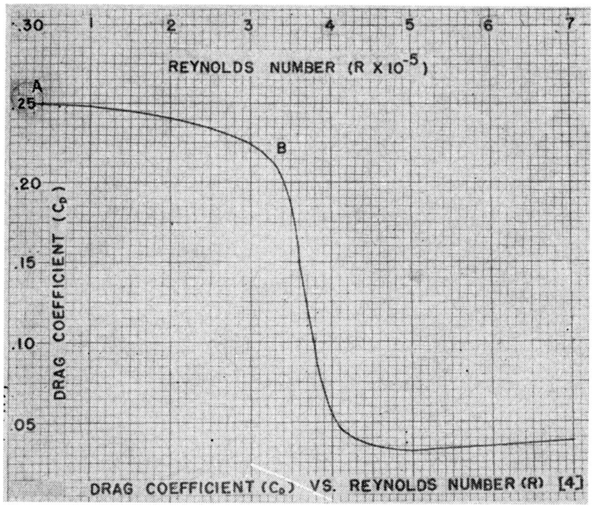

FIG. 1. Drag coefficient (C_D) *vs.* Reynolds Number (R) [5].

just sustain the fully grown hailstone (i.e. the stone's terminal velocity).

(2) that this updraft velocity is derived from the action of the buoyancy force acting on the parcels in the updraft above the level of free convection.

(3) that this velocity (and, therefore, the hail size) may be calculated from the positive area below the level of hail formation as determined on a thermodynamic diagram of a sounding of the air mass taken very close to the site and time of the hail occurrence, or of a sounding predicted for the site and time of the hail occurrence.

RELATING TERMINAL VELOCITY OF HAILSTONES TO THEIR DIAMETER

The work of Bilham and Relf [5] is relied upon to a major extent here, particularly their relationship between the coefficient of drag and Reynolds Number (FIGURE 1).

It is assumed that the hailstone is a sphere. This assumption is valid in the majority of hail occurrences; however, if the hailstone departs substantially from the shape of a sphere the following derived relationship is not expected to hold.

When dropped from rest in a quiescent atmosphere an object will accelerate downward until the aerodynamic drag force is just equal to the weight of the object. Its fall velocity under such a condition is termed its terminal velocity. If the atmosphere is moving upward relative to the object at exactly this velocity, the object will be maintained at rest in the vertical.

The analytic relationship is obtained from:

$$(1) \qquad \text{DRAG} = \text{WEIGHT}$$

It can be shown that

$$(2) \qquad w = \left(\frac{2\rho' dg}{3C_D \rho} \right)^{\frac{1}{2}}$$

where w is the vertical velocity of air (positive upward), ρ' is the density of the hailstone, d is the diameter of the hailstone, g is the acceleration of gravity (980 cm/sec^2), C_D is the coefficient of drag as used by Bilham and Relf [5] and ρ is the density of air.

The problem of ascertaining values for the coefficient of drag has been the subject of many investigations. Bilham and Relf [5] used values for C_D obtained from observations of spheres towed by aircraft. FIGURE 1 shows the relationship they found between drag coefficient and Reynolds number as given in their paper [5]. The region A–B on this curve lies within meteorologically probable limits of updraft velocities in nature. Bilham and Relf state that the sharp decrease in C_D beyond B suggests a limiting size of hailstones at nearly five inches in diameter.

Reynolds number is defined:

$$(3) \qquad R = \frac{\rho w d}{\mu} \quad \text{or} \quad w = \frac{R\mu}{\rho d}$$

where μ is the coefficient of viscosity.

Substituting for w in (2) from (3):

$$(4) \qquad d = \left(\frac{3R^2\mu^2 C_D}{2\rho' g \rho} \right)^{\frac{1}{3}}.$$

If values for R and C_D are selected from FIGURE 1 from the region from A to B, a number of values of d may be obtained. It is then possible to solve for corresponding values of w by substituting these d values in (3).

VERTICAL VELOCITY FROM BUOYANT ACCELERATION

The fundamental relationship involved here is the buoyant force, specifically stated:

$$(5) \qquad \dot{w} = g\left(\frac{T' - T}{T} \right) = g\frac{\Delta T}{T}$$

where $\dot{w}$ is the vertical acceleration of a parcel (positive upward), T' is the temperature of the parcel and T the temperature of the environment. Assuming steady state conditions (not unreasonable in an intense, established updraft):

$$(6) \qquad w\frac{\partial w}{\partial z} = g\frac{\Delta T}{T}$$

where z is height. And, for integration:

$$w \delta w = g \frac{\Delta T}{T} \delta z.$$

In order to integrate the above equation let $\Delta T = Kz$ where z is measured from the level of free convection upward. Also, at the level of free convection it is assumed $w = 0$. The integration is performed to height, H, measured from the level of free convection upward. Height, H, is the level of hail formation, however determined. The approximation is made $T = T_m =$ constant (T_m being the mean temperature of the environment from the level of free convection through height H).

$$(7) \qquad \int_0^{w_H} w \delta w = \frac{Kg}{T_m} \int_0^H z \delta z$$

$$(8) \qquad w_H^2 = \frac{Kg}{T_m} H^2$$

$$(9) \qquad w_H = \left(\frac{g}{T_m} \Delta T_H H \right)^{\frac{1}{2}}$$

It is now possible to substitute values of w from equation (3) as w_H in equation (9) to solve for $\Delta T_H H$. Assigning values to H, corresponding values for ΔT_H are obtained. In other words, hailstone diameter has been related to the positive area analyzed for a sounding plotted on a thermodynamic diagram. In U. S. Weather Bureau practice, this diagram is the pseudo-adiabatic diagram (Form WB 1147). Height is measured in terms of the "standard atmosphere."

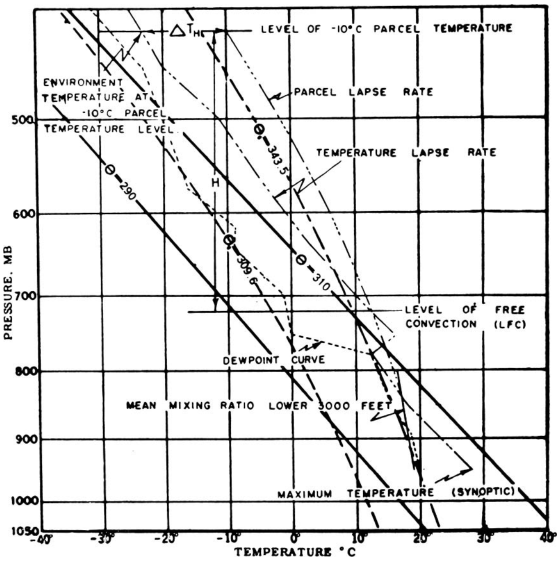

Fig. 2. Sounding for Altus, Oklahoma, 1500C, 29 April 1954. Hail up to 2″ in diameter fell in Dewey County, about 95 miles north of Altus, between 1530C and 1830C. There were many reports of smaller hail in western Oklahoma during the evening.

The Positive Area

The next problem is that of determining the dimensions of a representative positive area. Where is the level of free convection? Where is the level of hail formation?

Experience has shown that a fairly representative level of free convection can be found as follows:

(1) average the mixing ratio and potential temperature values in the lower 3000 feet of a sounding representative of the environment just prior to thunderstorm activity;

(2) lift these values upward along their respective isopleths until saturation is reached;

(3) continue upward along the pseudo-adiabat through this point of saturation to the point of intersection with the temperature lapse rate curve. This is the level of free convection used here.

H is the height of the level of hail formation above the level of free convection. This level is visualized as a mean plane for a layer of some thickness, probably appreciable in most cases. Reference [6] states that "temperatures inside hailstones, determined immediately after the stones strike the ground, have been found to be at, or less than 0°C, and often between − 5°C and − 15°C. . . ." Meeker's [7] hailstorm model indicates that a mean temperature for the region of hail formation is − 15°C or lower. It was decided to try a mean figure of the above values of − 10°C as representative of the temperature at the level of hail formation. It was further decided that, since in the basic hypothesis the hail forms in the thunderstorm updraft, the choice should be the − 10°C level on the predicted parcel lapse rate rather than an environment temperature of − 10°C.

To find H, then, continue up the pseudo-adiabat through the level of free convection to the parcel temperature of − 10°C. The distance between the level of free convection and this point is H. The algebraic difference between − 10°C and the environment temperature at the − 10°C parcel temperature level is ΔT_H. For this to be significant, the parcel must, of course, be warmer than the environment. The foregoing procedure is illustrated in Figure 2.

The approximation to a triangular area has been made in determining the positive area. Occasion-

TABLE 1

THE RELATIONSHIP BETWEEN HAILSTONE DIAMETER AND TERMINAL VELOCITY AT VARIOUS LEVELS DUE TO THE EFFECTS OF THE VARIATION IN AIR DENSITY (A TEMPERATURE OF −10°C IS USED)

Diameter (Inches)	Pressure, mb		
	400	500	600
	Speed (Ft./Min)		
$\frac{1}{4}$	2,913	2,631	2,373
$\frac{1}{2}$	4,099	3,678	3,359
$\frac{3}{4}$	5,014	4,491	4,113
1	5,796	5,180	4,725
$1\frac{1}{2}$	7,124	6,370	5,817
2	8,240	7,380	6,742
$2\frac{1}{2}$	9,242	8,279	7,563
3	10,157	9,133	8,317
$3\frac{1}{2}$	11,009	9,907	9,092
4	11,902	10,787	9,868
$4\frac{1}{2}$	12,790	11,558	10,645
5	13,680	12,507	—

ally in those cases of irregular temperature lapse rates this approximation may be improved by determining the mean lapse rate from the level of free convection through height H and using this to determine ΔT_H. This may be done by the graphical equi-areal method.

A positive area calculation can be handled more precisely on true energy area diagrams by the direct equating of the kinetic energy of the rising parcel to the energy indicated. However, the percentage of error arising in the above alternative approach would not vary the final results beyond limits otherwise imposed, and the results are directly utilizable on current working diagrams.

It is noted that entrainment has been neglected here. In the case of the intense thunderstorms which produce the larger hail sizes, the parcels may be affected very little by entrainment at the core of strong updrafts of large cross-section. Entrainment may well account for the spectrum of hail sizes observed over an area due to a spectrum of entrainment rates in the complexity of thunderstorm cells. Undoubtedly, entrainment must account for a goodly number of the failures experienced with any hail forecasting technique that cannot account for its effects. However, this physically based technique permits an extension to include in some manner this process at some future date. In the meantime, satisfactory results may be obtained in the prediction of maximum hail sizes even though this factor is neglected.

COMPUTATIONS AND PREPARATION OF GRAPHS

Values for C_D and R were extracted from the graph in FIGURE 1.

It was noted that the variation in air density with the pressure of the −10°C level would significantly affect the computed hailstone sizes; therefore, computations were carried out for 400, 500 and 600 mb.

A hailstone density of 0.7 grams per cubic centimeter was chosen. There has been considerable variation noted in densities assigned by various writers. This value seemed a good median, and support is found in [6] for its selection.

Some other values used are:

$$\mu = 1.665 \times 10^{-4} \text{ gr cm}^{-1} \text{ sec}^{-1} \text{ for air at } -10°\text{C}.$$

$$\rho = 0.530 \times 10^{-3} \text{ gr cm}^{-3} \quad \text{for air at } -10°\text{C at 400 mb.}$$

$$\rho = 0.622 \times 10^{-3} \text{ gr cm}^{-3} \quad \text{for air at } -10°\text{C at 500 mb.}$$

$$\rho = 0.795 \times 10^{-3} \text{ gr cm}^{-3} \quad \text{for air at } -10°\text{C at 600 mb.}$$

With these values, hailstone diameters and corresponding terminal velocities were computed from equations (3) and (4). The results of these computations are shown in TABLE 1.

The next step was the solution for ΔT_H at the various levels by substitution of appropriate w

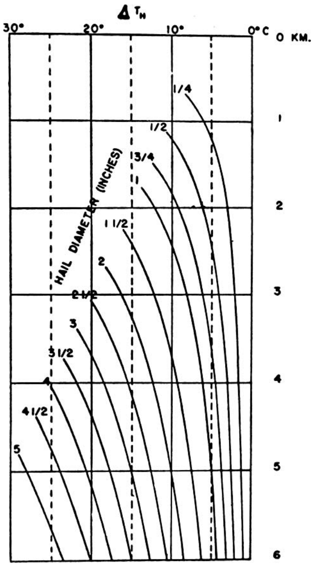

FIG. 3. Foster/Bates hail size diagram. Sizes shown are for −10°C parcel temperature at 400 mb. Correct from FIGURE 4 for other levels. Diagram overlays WB Form 1147.

from Table 1 and assigning various values of H. (Though the variation was not too significant, T_m was varied from 266° for H of one kilometer to 278 for H of six kilometers.) The results of this computation for the 400 mb level are shown in Table 2.

The above calculation was also carried out for the 500 and 600 mb levels. However, rather than involve three graphs in the technique with an attendant interpolation in most cases, it is nomographically preferable to use a graph at one level, chosen as 400 mb, and correct the sizes obtained on one additional graph constructed from points from the graphs of the three levels.

The graph for hail size at 400 mb is shown in Figure 3. It is constructed on coordinates of WB Form 1147 in order to utilize the hail size isopleths on a transparent overlay for rapid calculation. The correction graph is shown in Figure 4. It was deemed within the limits of accuracy to extend the curves to 650 mb. While it is very unlikely the dimensions of a positive area will ever exceed a ΔT_H of 15° and an H of 5 kilometers, with a corresponding hail size of three inches, the tables and graphs were carried out to values representing a dry adiabatic lapse rate of temperature from a level of free convection near sea level to a -10°C lifted parcel temperature near 400 mb.

In practice, the graph in Figure 3, as an overlay, is placed with the zero point coincident with the -10°C point on the parcel lapse rate and the coordinates parallel with the appropriate coordinates on the WB 1147 base. Hail size at 400 mb is read at the intersection of the environment temperature isotherm at the -10°C parcel temperature level with the level of free convection.

If the level of the -10°C parcel temperature is different from 400 mb, the hail size thus obtained and the actual pressure level are used to enter the graph in Figure 4 to obtain the corrected hail size. It will be noted that the higher the pressure of this level, the greater the hail size.

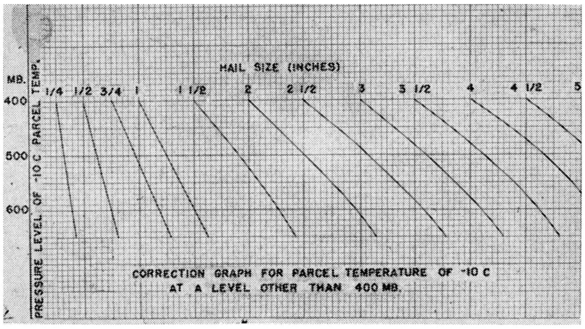

Fig. 4. Correction graph for parcel temperature of -10°C at a level other than 400 mb.

Table 2

Values of ΔT_H in °C for Various Hailstone Diameters at Selected Geometric Values of H. Values are Shown for a -10°C Parcel Temperature Level Coincident with 400 mb.

Diameter (Inches)	ΔT_H(°C) *vs.* H (*Km*)					
	1	2	3	4	5	6
$\frac{1}{4}$	6.0°	3.0°	2.0°	1.5°	1.2°	1.1°
$\frac{1}{2}$	11.9°	6.0°	4.0°	3.0°	2.5°	2.1°
$\frac{3}{4}$	—	9.0°	6.1°	4.6°	3.7°	3.1°
1	—	12.0°	8.1°	6.1°	4.9°	4.2°
$1\frac{1}{2}$	—	18.1°	12.1°	9.2°	7.4°	6.2°
2	—	—	16.2°	12.3°	9.9°	8.3°
$2\frac{1}{2}$	—	—	20.4°	15.4°	12.4°	10.5°
3	—	—	—	18.6°	15.0°	12.6°
$3\frac{1}{2}$	—	—	—	21.9°	17.6°	14.8°
4	—	—	—	25.3°	20.6°	17.3°
$4\frac{1}{2}$	—	—	—	—	23.8°	20.0°
5	—	—	—	—	27.9°	23.5°

Testing the Technique

Climatological records of the U. S. Weather Bureau were screened for reports of hail which included size, time and location for the months April through August of 1953 and 1954. In order to secure a reasonable number of reports, it was necessary to assign diameters in inches to descriptive reports. Some values used were:

Marbles	$\frac{1}{2}$ inch
Walnuts	$1\frac{1}{2}$ inches
Golf balls	$1\frac{3}{4}$ inches
Hen eggs	2 inches
Tennis balls	$2\frac{1}{2}$ inches
Baseballs	$2\frac{3}{4}$ inches
Teacups	3 inches

Due to subjectivity, this, of course, injected an error interval into the data; however, no alternative procedure presented itself.

Soundings closest in time preceding the hail occurrence and closest in space were then selected. Surface charts synoptic with these soundings were checked, and non-representative soundings discarded, e.g., those through frontal surfaces, those with sharp moisture discontinuities between the station and the site of hail occurrence, and those with squall lines which had already passed the station and lay between the station and the site of hail occurrence.

In some cases, modification of the surface point was made from surrounding surface data to more representative temperature and dew point for the

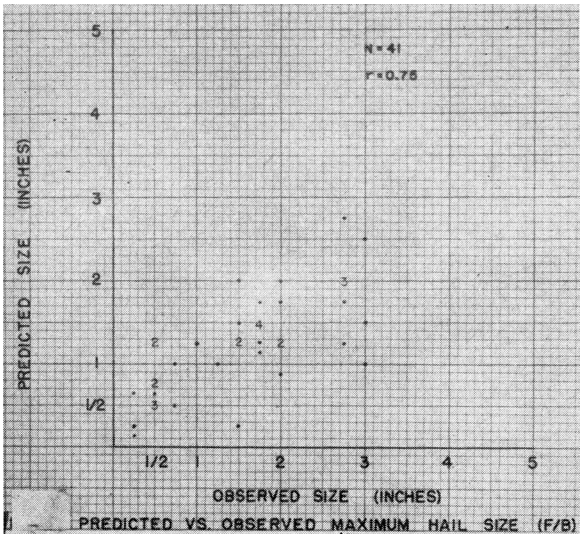

Fig. 5. Predicted *vs.* observed maximum hail size (Foster/Bates method).

site of hail occurrence. Only the surface point of the sounding was changed. In no instance did these modifications disturb the value of the soundings as comparative cases.

Forty-one cases were finally selected where hail was reported within three hours after release time of the radiosonde and within sixty miles of the radiosonde station. These soundings were analyzed and hail size predicted by both the Fawbush-Miller method [1] and the Foster-Bates technique. The results obtained in this sampling are shown in Figures 5 and 6. The correlations obtained are within the limits of statistical significance, and the above described technique shows a slight improvement.

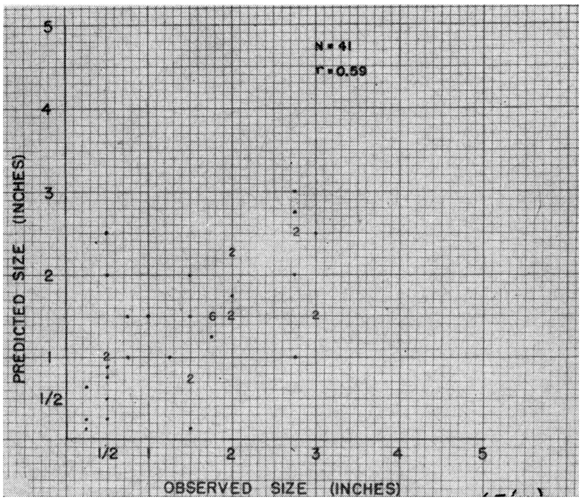

Fig. 6. Predicted *vs.* observed maximum hail size (Fawbush/Miller method).

To date the Weather Bureau's SELS Center has not tried to predict hail of any specified size, but has predicted hail above or below a limit of ¾ inches in diameter. The probability of the success of the two systems are compared in Table 3.

It should be understood that the probabilities of success quoted are not for the *occurrence* of hail. These figures state that, given a hail producing thunderstorm, the probability of accurately forecasting the hail within two size classes is as shown. The forecast of the occurrence of hail, while not independent of the available potential energy, must involve other, more complex considerations. Consequently, the probability of success in forecasting the occurrence *and* the hail size resulting will be somewhat less than the figures show.

Table 3

Evaluation of Predictions by Classes About a ¾ Inch Size

Observed	Forecast		Foster/Bates method
	$<\frac{3}{4}$ in.	$\geq\frac{3}{4}$ in.	
$<\frac{3}{4}$	7	4	No. successes in predicting correct class = 35
$\geq\frac{3}{4}$	2	28	% probability of success = 85%
Observed	**Forecast**		**Fawbush/Miller method**
	$<\frac{3}{4}$ in.	$\geq\frac{3}{4}$ in.	
$<\frac{3}{4}$	5	6	No. successes in predicting correct class = 34
$\geq\frac{3}{4}$	1	29	% probability of success = 83%

Concluding Remarks

The treatment of an extremely complex problem has not been thoroughly exhausted here, and no pretense to a final solution is intended. Given the basic validity of the relationship of hail size to updraft velocities, the necessity for the selection of mean values and approximations to carry the relationship through to positive areas, the problem of obtaining representative dimensions of positive areas as related to air mass modification not accounted for, entrainment effects, and the problem of procurement of good samples for testing and verification broadens the margin of error. Further improvement in the results can be expected with a more precise treatment of any of these.

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NEWS AND NOTES

World Meteorological Organization Reviews Year of Active Assistance

From utilization of wind power in Haiti to locust control in British East Africa, the World Meteorological Organization continues to aid governments in devising or employing weather data to meet the needs and the problems of many areas of the world. One of ten specialized agencies which are autonomous bodies but which cooperate with the United Nations under special agreements, WMO shares in the United Nations Expanded Program of Technical Assistance. This is financed by a special fund of voluntary contributions pledged by 70 countries.

Under the Expanded Program, World Meteorological Organization provided 22 missions and 31 fellowships to 23 countries in 1955 to help them establish or develop national weather services.

In Afghanistan and Nicaragua, the organization's aid last year was directed at helping these countries start their first meteorological services of any kind. Another mission was sent to Iran to assist in coordinating various weather services leading to the establishment of a National Meteorological Institute.

The development of WMO's technical assistance activity is shown by the fact that approximately $300,000 was expended during the year for this purpose, as compared with $50,000 in 1953. Projects already approved for 1956 will cost $420,000.

Value to Agriculture

Stressing the value of weather forecasting in warning farmers of possible plant diseases, WMO sent a specialist to Chile to advise the government on ways in which the science of weather may help to control the menacing potato blight in that country. By predicting the arrival of weather favorable to the growth of disease, meteorology may enable the farmer to take preventive action, such as applying fungicide, in time to protect his crops.

Similarly, a World Meteorological Organization expert was engaged in drafting a memorandum on a proposed meteorological desert locust warning system in Africa, and in studying ways of obtaining a better understanding of the relationship between locusts and weather.

Another field of meteorological activity of this agency in 1955 concerned its work on world maps showing areas of thunderstorm activity based on observations made on ships and land.

A new WMO code for transmitting weather messages was introduced on January 1, 1955, for use in some 1200 broadcasts and transmissions in which 110,000 daily weather observations for exchange between nations and continents are disseminated day and night, almost without interruption.

The possibility of utilizing wind for power was studied during the past year by a WMO expert sent to Haiti on request of that government.

Congress Action

The WMO Congress met for the first time since 1951 in Geneva in April 1955, and decided to continue to support actively the Arid Zone Research Program undertaken in collaboration with UNESCO. This program has numerous meteorological aspects, including the study of artificial rain, wind power, and solar energy. It also took steps to join with UNESCO in planning a program of research regarding the humid tropics. Furthermore, the Congress agreed to take the responsibility for international hydro-meteorological activities relating to the development and control of water resources.

In addition, the organization sent representatives to the first United Nations Conference on the Peaceful Uses of Atomic Energy, held in Geneva in August.

WMO continued its regular functions during the past year in assisting countries in meeting the need for an uninterrupted international exchange of weather information; a worldwide standardization of the techniques and instruments for weather observations; and safety of international travel and transport through a coordinated universal system of weather reporting, forecasting, and warning for international aviation and shipping.

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